Digital Filter Design (FIR) Using Frequency Sampling Method

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ABSTRACT:

The work reported in this paper deals with a finite impulse response FIR digital filter design using frequency sampling method. The frequency sampling method allows us to design recursive and non-recursive FIR filters for both standard frequency selective and filters with arbitrary frequency response. There are two distinct types of frequency sampling filters, depending on where the initial frequency sample occurred

The digital filter is used to filter discrete time signals with the ability to modify the frequency response of the filter at any time and is used in many application such as data

compression, biomedical signal processing, communication receivers, etc.

keywords. *FIR filters , Digital filter .*

Introduction :

A filter is essentially a system or a network that selectively changes the wave shape, amplitude–frequency and/or phase-frequency characteristics of a signal in desired manner. Common filtering objectives are to improve the quality of a signal, to extract information from signal or to separate two or more signals previously combined. A digital filter is mathematical algorithm implemented in hardware and/or software that operates on a digital input signal to produce a digital output signal for the purpose of achieving a filtering objective. Digital filters often operate on digitized analog signals or just numbers, representing some variable, stored in a computer memory. A simplified block diagram of real–time digital filter, with analog input and output signals ,is given in Figure 1 the band limited analog signal is sampled periodically and converted into a series of digital samples, x(n), n=0,1,2,………. The digital processor implements the filtering operation, mapping the input sequence, x (n) , into the output sequence, y(n) , in accordance with a computation algorithm for the filter. The DAC converts the digitally filtered output analog values which are then analog filtered to smooth and remove unwanted high frequency components. [1] , [2]

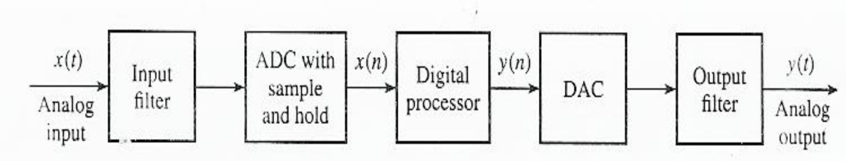

Figure 1 A simplified block diagram of a real-time digital filter.

Digital filters types:

Digital filters are divided into two classes, infinite impulse response (IIR) and finite impulse response (FIR) filters. The input and the output signals are related by

the convolution sum, which is given below:

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \quad \text{for the IIR filter} \qquad (1)$$

and

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \quad \text{for the FIR filter} \qquad (2)$$

where h (k) the impulse response sequence h (k), k = 0,1,…….., from these equations we see that, for IIR filters, the impulse response is of infinite duration while for FIR filters, the impulse response is of finite duration, h (k) has only N values. In practical, we can not compute the output of the IIR filter using equation (3.1) because the length of its impulse response is too long, in stead, the IIR filtering equation is expressed in a recursive form

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=1}^{N} a_k y(n-k) \qquad (3)$$

where the a_k and b_k are the coefficients of the filter. From equation (3) we note that, the current output sample, y (n) is a function of past values of output and the present and past input samples, that is the a feedback system of some sort. the equation (3) reduces to the FIR equation when the b_k are set to zero and we note that in the FIR filter current output sample, y(n) is a function only of past and present values of input sample.[3]

The transfer functions of FIR and IIR filters are given in the equations (4) and (5) respectively which very useful equations in evaluating their frequency responses.

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} \quad (4)$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad (5)$$

In the design of frequency –selective filters, the desired filter characteristics are specified in the frequency domain in terms of the desired magnitude and phase response of the filter. In the filter design process, we determine the coefficients of a causal FIR, IIR filter that closely approximates the desired frequency responses specification. The issue of which type of filter to design, FIR or IIR, depends on the nature of the problem and on the specification of the desired frequency responses. [5]

Filter Specification :

Requirement specification include specifying signal characteristics as types of signal source, data rates, filter characteristics as type of filter (for example low pass filter), the desired amplitude and / or phase responses and their tolerances and speed of operation the sampling frequency, and other design constraints as the cost of the filter design. These requirements often dependent on the application of the digital filter. [7]

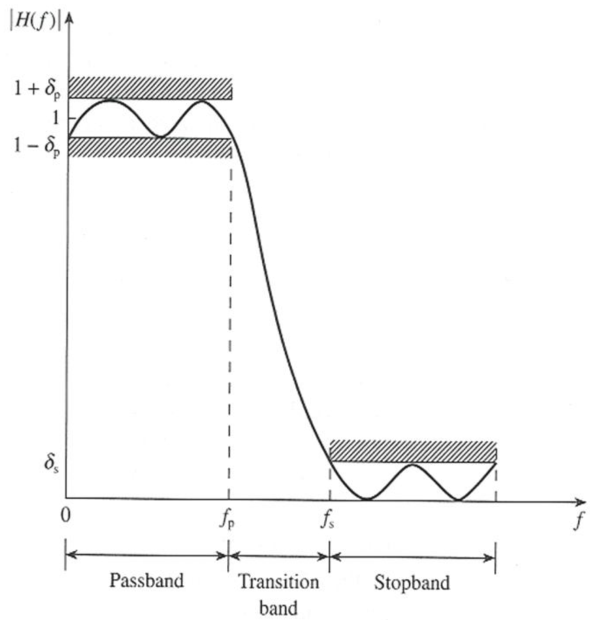

Figure 2 Magnitude-frequency response specification for a low pass filter.

The characteristics of digital filters are often specified in the frequency domain. The amplitude – frequency response of FIR and IIR filter is often specified in the form of a tolerance scheme as given in the figure (2). Referring to the figure above, the following parameters are of interest:

δ_p peak pass band deviation (or ripples)
δ_s stop band deviation
f_p pass band edge frequency
f_s stop band edge frequency
F_S sampling frequency

pass band and stop band deviation may be expressed as ordinary numbers or in decibels. The minimum stop band attenuation, A_s, and the peak pass band ripple, A_p in decibels are given by:

$$A_s \text{ (stop band attenuation)} = -20 \log_{10} \delta_s \qquad (6)$$

$$A_P \text{ (pass band ripple)} = 20 \log_{10} (1+\delta_p) \qquad (7)$$

The edge frequencies f_p and f_s often given in normalized form, that is as a function of the sampling frequency (f/F_S), the difference between f_s and f_p gives the transition width of the filter, another important parameter is the filter length, N, which defines the number of filter coefficients given, and if the filter length N go to the infinity the filter will closely to the ideal case.[4]

Frequency sampling method:

The frequency sampling method allows us to design recursive and nonrecursive FIR filters for both standard frequency selective and filters with arbitrary frequency response.

A. No recursive frequency sampling filters :

The problem of FIR filter design is to find a finite–length impulse response h (n) that corresponds to desired frequency response. In this method h (n) can be determined by uniformly sampling, the desired frequency response $H_D(\omega)$ at the N points and finding its inverse DFT of the frequency samples as shown in the Figure 4.

$$h(n)=\frac{1}{N}\sum_{k=0}^{N-1}H(k)e^{j(\frac{2\pi}{N})nk} \qquad (8)$$

where H (k), k = 0, 1, 2,……., N-1, are samples of the $H_D(\omega)$.

For linear phase filters, with positive symmetrical impulse response, we can write

$$h(n)=\frac{1}{N}\left[\sum_{k=1}^{(N/2)-1}2\left|H(k)\right|\cos\left[\frac{2\pi k(n-\alpha)}{N}\right]+H(0)\right] \qquad (9)$$

where α = (N-1)/2. For N odd, the upper limit in the summation is (N - 1)/2, to obtain a good approximation to the desired frequency response, we must take a sufficient number of the frequency samples [9].

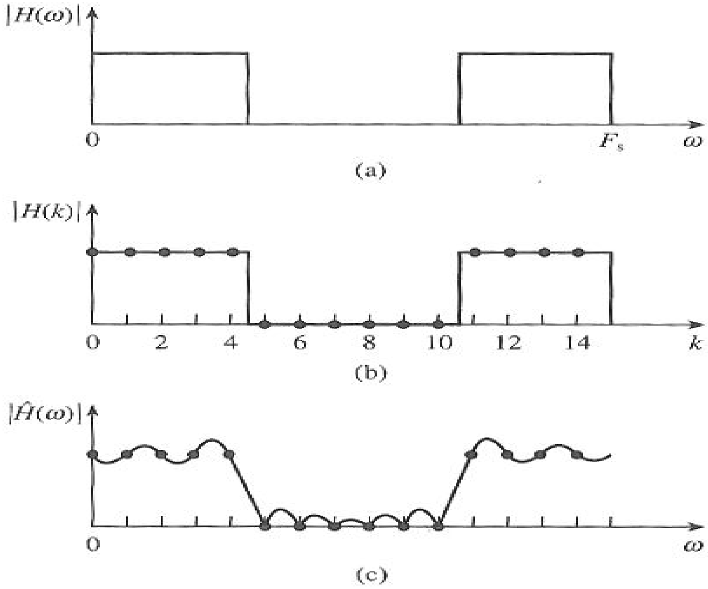

Figure 3 (a) Frequency response of an ideal lowpass filter. (b) Samples of the ideal lowpass filter. (c) Frequency response of lowpass filter derived from the frequency samples of (b).

B. Types 1 and 2 frequency sampling filters :

frequency sampling filters are based on specification of a set of samples of the desired frequency response at N uniformly spaced points around the unit circle. The set of frequencies that have been used until this point is determined by the relation.

$$f_k = \frac{k}{N} F_S, \quad k = 0,1,....,N-1 \qquad (10)$$

Corresponding to the N frequencies at which an N-point DFT is evaluated. There is a second set of uniformly spaced frequencies for which a frequency sampling structure can conveniently be obtained. This set of frequencies determined by the relation [6].

$$f_k = \frac{(k+1/2)}{N} F_S, \quad k = 0,1,....,N-1 \qquad (11)$$

Figure (4) shows exactly where the frequency sampling points are located for the two sets of frequencies the first set of frequencies in equation (10) is called type 1, while the second set in equation (11) is called type 2 for the case where N is both even or odd. The type 1 designs have the initial point at f = 0, where as the type 2 designs have the initial point at f = 1/2N.

The importance of type 2 frequency samples lays the additional flexibility it gives the design method to specify the desired frequency response at a second possible set of frequencies

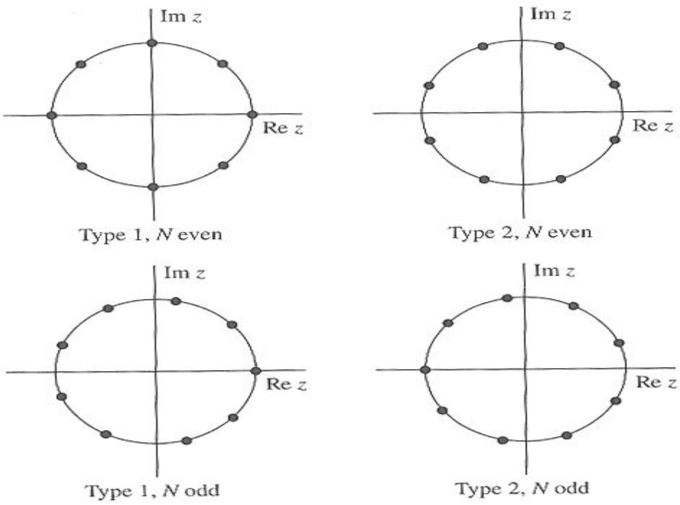

Figure 4 Two type of the frequency sampling filter.

C. Recursive frequency sampling filter :

In recursive frequency sampling method the DFT samples H (k) for an FIR sequence can be regarded as samples of the filters z–transform, evaluated at N points equally spaced around the unit circle.[8]

$$H(k) = H(z)\big|_{z=e^{j(2\pi/N)k}} \qquad (12)$$

thus the z–transform of an FIR filter can easily be expressed in terms of its DFT coefficients,

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{j(2\pi/N)nk}\right] z^{-n}$$

$$= \sum_{k=0}^{N-1} \frac{H(k)}{N} \sum_{n=0}^{N-1} \left[e^{j(2\pi/N)k} z^{-1} \right]^n$$

$$= \sum_{k=0}^{N-1} \frac{H(k)}{N} \frac{(1 - e^{j2\pi k} z^N)}{(1 - e^{j(2\pi/N)k} z^{-1})} \qquad (13)$$

by butting $e^{j2\pi k} = 1$, Equation (13) reduces to

$$H(z) = \frac{(1 - z^{-N})}{N} \sum_{k=0}^{N-1} \frac{H(k)}{(1 - z^{-1} e^{j(2\pi/N)k})} \qquad (14)$$

This is the desired result

Conclusion :

The main idea of the frequency sampling design method is that a desired frequency response can be approximated by sampling it of N evenly spaced points and then obtaining an interpolated frequency response that passes through the frequency samples. For filters with responably smooth frequency responses. The interpolation error is generally small. In the case of band select filters, where the desired frequency response changes radically across bands, the frequency samples which occur in transition bands are made to be unspecified variables whose values are chosen by an optimization algorithm which minimizes some function of the approximation error of the filter finally, it was shown that there were two distinct types of frequency sampling filters, depending on where the initial frequency sample occurred [6].

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